Reading 5 (Due Friday 8/8/25 by 5PM)

Directions: In Calc I, a differentiable functions is a function that is locally linear i.e. can be approximated by a linear function (a tangent line) at every point. In Chapter 10, we will work towrds defining what it means for a function to be *continuously differentiable*. A function that is continuously differentiable can be approximated by linear functions (a tangent plane) at every point. In order to define *continuity* we need to define *limit*. Your job is to come to class with some understanding of what the limit of a multi-variable function is.

Read the following sections of the text:

- All of Section 10.1. **I will not be lecturing on 10.1, but we will do a reading debrief.**

and complete the following tasks along the way. If an Activity is not listed, you do not need to complete it (although you are welcome to read it). Turn your write up in via gradescope. You do not need to write the questions down, as long as you clearly indicate the question number.

1. Preview Activity 10.1.1.

2. Activity 10.1.2.

3. Activity 10.1.3.

4. Compute the following limits using the properties of limits. Justify your reasoning. Compare with the associated graph.

(a) $\lim_{(x,y)\to(\pi,\pi/2)} y\sin(x-y)$. The graph: GeoGebra: Reading 5 Task 4(a).

(b) $\lim_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$. The graph: GeoGebra: Reading 5 Task 4(b).

5. Use appropriate limit along different paths to show that the following limits do not exist. Justify your reasoning. Compare with the associated graph.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{xy^4}{x^2+y^8}$. GeoGebra can't accurately graph this one near the limit.

(b) $\lim_{(x,y)\to(0,0)} f(x,y) = \dfrac{3x^2}{3x^2+5y^2}$. The graph: GeoGebra: Reading 5 Task 5(b)

6. Use the properties of continuity to determined the set of points for which the following functions are continuous. Justify your reasoning. Compare with the included graph.

(a) $f(x,y) = \dfrac{1}{9-x^2-y^2}$. The graph: GeoGebra: Reading 5 Task 6a. Note: computer generated graphs exhibit strange behavior near some discontinuities!

(b) $f(x,y) = \dfrac{e^x+e^y}{e^{xy}-1}$. The graph: GeoGebra: Reading 5 Task 6b.

7. After reading all of Section 10.1, write down three (or more) things you learned or still have questions about.

For your reference, the correct answers are included in the footnote. on the next page[1]

Basic learning objectives: These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated in italics.

1. Compute *limits* of multi-variable functions using Properties of Limits. Use appropriate methods to prove that a limit does not exist.
2. Determine whether a function is continuous at a point using various methods.
3. Understand how the level curves of a multivariable function can provide insight into whether a limit does or does not exist.

Advanced learning objectives: In addition to mastering the basic objectives, here are the tasks you should be able to perform **after class, with sufficient practice:** (these learning objectives are for Section 10.2, which is not part of the reading)

1. Compute partial derivatives of multivariable functions using only the limit definition.
2. Compute partial derivatives using the differentiation laws from calculus of a single variable.
3. Interpret the first partial derivatives $f_x(x,y)$ and $f_y(x,y)$ geometrically in relation to the graph of the function. Interpret the first partial derivatives as a rate of change.
4. Estimate partial derivatives given a table of finitely many values of a function, or a contour plot of a function.

[1] 4(a) $\lim_{(x,y)\to(\pi,\pi/2)} y\sin(x-y) = \pi/2$,
4(b) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = 2$
5(a) approach $(0,0)$ along the paths $r_1(t) = (t,0)$ (the x-axis) and $r_2(t) = (t, t^{1/4})$ (the graph of $f(x) = x^{1/4}$ in the x,y-plane).
5(b) approach $(0,0)$ along the x-axis and the y-axis.
6(a) Every point in $\mathbb{R}^2$ except points on circle of radius 3 centered at the origin.
6(b) every point in $\mathbb{R}^2$ except points on the line $x=0$ or the line $y=0$.